INSTANT SUPPLEMENTARY EXAM QUESTION PAPER

JULY - 2016 (With Answers)

STD. X - MATHEMATICS

[Time Allowed : 2½ Hrs.] **[Maximum Marks : 100]**

SECTION - I

Note: (*i*) Answer **all** the **15** questions.

(*ii*) Choose the **correct** answer from the given **four** alternatives and write the option code and the corresponding answer. **[15 × 1 = 15]**

1. If $f(x) = x^2 + 5$, then $f(-4) =$
   (a) 26 (b) 21 (c) 20 (d) – 20

2. If a, b, c are in G.P., then $\frac{a-b}{b-c}$ is equal to :
   (a) $\frac{a}{b}$ (b) $\frac{b}{a}$ (c) $\frac{a}{c}$ (d) $\frac{c}{b}$

3. If a, b, c, l, m are in A.P., then the value of $a - 4b + 6c - 4l + m$ is :
   (a) 1 (b) 2 (c) 3 (d) 0

4. If the system $6x - 2y = 3$, $kx - y = 2$ has a unique solution, then :
   (a) $k = 3$ (b) $k \neq 3$
   (c) $k = 4$ (d) $k \neq 4$

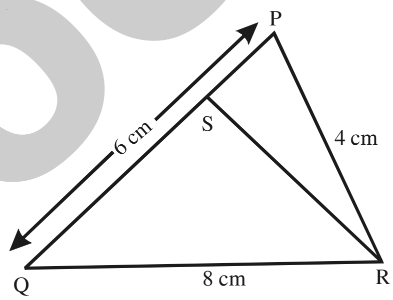
5. The square root of $49(x^2 - 2xy + y^2)^2$ is :
   (a) $7|x-y|$ (b) $7(x+y)(x-y)$
   (c) $7(x+y)^2$ (d) $7(x-y)^2$

6. If $(5 \; x \; 1)\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = (20)$, then the value of x is :
   (a) 7 (b) –7 (c) $\frac{1}{7}$ (d) 0

7. The centre of a circle is (–6, 4). If one end of the diameter of the circle is at (–12, 8) then the other end is at :
   (a) (–18, 12) (b) (–9, 6)
   (c) (–3, 2) (d) (0, 0)

8. Area of the triangle formed by the points (0, 0), (2, 0) and (0, 2) is :
   (a) 1 sq. units (b) 2 sq. units
   (c) 4 sq. units (d) 8 sq. units

9. In ΔPQR, RS is the bisector of $\angle R$. If PQ = 6 cm, QR = 8 cm, RP = 4 cm then PS is equal to :



   (a) 2 cm (b) 4 cm
   (c) 3 cm (d) 6 cm

10. ΔABC is a right angled triangle where $\angle B = 90^\circ$ and $BD \perp AC$. If BD = 8 cm, AD = 4 cm, then CD is :
    (a) 24 cm (b) 16 cm
    (c) 32 cm (d) 8 cm

11. $(\cos^2\theta - 1)(\cos^2\theta + 1) + 1 =$
    (a) 1 (b) –1 (c) 2 (d) 0

12. If $\tan\theta = \frac{a}{x}$, then the value of $\frac{x}{\sqrt{a^2+x^2}} =$
    (a) $\cos\theta$ (b) $\sin\theta$
    (c) $\operatorname{cosec}\theta$ (d) $\sec\theta$

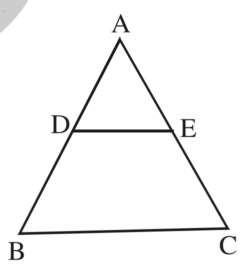
13. Curved surface area of solid sphere is 24 cm^2. If the sphere is divided into two hemispheres, then the total surface area of one of the hemisphere is :
    (a) 12 cm^2 (b) 8 cm^2
    (c) 16 cm^2 (d) 18 cm^2

14. If the standard deviation of a set of data is 1.6, then the variance is :
    (a) 0.4 (b) 2.56 (c) 1.96 (d) 0.04
15. Probability of sure event is :
    (a) 1 (b) 0 (c) 100 (d) 0.1

SECTION - II

Note: (*i*) Answer **10** questions
(*ii*) Question number **30** is **compulsory**. Select **any 9** questions from the first **14** questions. **[10 × 2 = 20]**

16. If A = {4, 6, 7, 8, 9}, B = {2, 4, 6} C = {1, 2, 3, 4, 5, 6} then find $A \cup (B \cap C)$.
17. Let A = {1, 2, 3, 4, 5} B = N and $f : A \to B$ be defined by $f(x) = x^2$. Find the range of f. Identify the type of function.
18. Find a quadratic polynomial if the sum and product of zeros of it are –4 and 3 respectively.
19. If –5 is a root of the quadratic equation $2x^2 + px - 15 = 0$, find the value of p.
20. Let $A = \begin{bmatrix} 3 & 2 \\ 5 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 8 & -1 \\ 4 & 3 \end{bmatrix}$. Find the matrix C if C = 2A + B.
21. If $A = \begin{bmatrix} 1 & 3 \\ 9 & -6 \end{bmatrix}$, then verify AI = IA = A, where I is the unit matrix of order 2.
22. Find the centroid of the triangle whose vertices are A (4, –6), B (3, –2) and C (5, 2).
23. In a given ΔABC, DE || BC. If BD = $x - 3$, AB = $2x$, CE = $x - 2$ and AC = $2x + 3$. Find x.



24. Prove the identity
    $(\sin^6\theta + \cos^6\theta) = 1 - 3\sin^2\theta\cos^2\theta$.
25. The angle of elevation of the top of a tower as seen by an observer is 30º. The observer is at a distance of $30\sqrt{3}$ m from the tower. If the eye level of the observer is 1.5 m above the ground level, then find the height of the tower.
26. The radii of two right circular cylinders are in the ratio 2 : 3. Find the ratio of their volumes if their heights are in the ratio 5 : 3.
27. If the circumference of the base of a solid right circular cone is 236 cm and its slant height is 12 cm, find its curved surface area.
28. Calculate the standard deviation of the first 13 natural numbers.
29. Three dice are thrown simultaneously. Find the probability of getting the same number on all the three dice.
30. a) Find the sum of the following series.
    31 + 33 + ... + 53

    [OR]

    b) Find the equation of the line intersecting the y-axis at a distance of 3 units above the origin and $\tan\theta = \frac{1}{2}$, where θ is the angle of inclination.

SECTION - III

Note: (*i*) Answer **9** questions.
(*ii*) Question No. **45** is **Compulsory**. Select **any 8** questions from the first **14** questions. **[9 × 5 = 45]**

31. Using Venn diagram, verify the De-Morgan's Law of set difference
    $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$
32. Let A = { 0, 1, 2, 3} and B = {1, 3, 5, 7, 9} be two sets. Let $f : A \to B$ a function given by $f(x) = 2x + 1$. Represent this function as (i) a set ordered pairs (ii) a table (iii) an arrow diagram and (iv) a graph.

33. Find the sum of all natural numbers between 300 and 500 which are divisible by 11.

34. If the 4th and 7th terms of a G.P. are 54 and 1458 respectively, find the G.P.

35. Factorize : $x^3 - 3x^2 - 10x + 24$.

36. Find the values of a and b if the following polynomial is a perfect square.

$4x^4 - 12x^3 + 37x^2 + ax + b$.

37. If $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ then show that

$A^2 - 4A + 5I_2 = 0$.

38. The vertices of ΔABC are A (1, 8), B (–2, 4), C (8, –5). If M and N are the midpoints of AB and AC respectively, find the slope of MN and hence verify that MN is parallel to BC.

39. Find the area of the triangle formed by the line $4x + 3y - 12 = 0$ with the co-ordinate axes.

40. A vertical tree is broken by the wind. The top of the tree touches the ground and makes an angle 30° with it. If the top of the tree touches the ground 30 m away from its foot, then find the actual height of the tree.

41. A cricket stump is in the shape of a cylinder surmounted by a cone. The diameter of the cyclinder and the total height of the stump are 10 cm and 80 cm respectively. If the height of the conical part is 12 cm, then find its total surface area.

42. A spherical solid material of radius 18 cm is melted and recast into three small solid spherical spheres of different sizes. If the radii of two spheres are 2 cm and 12 cm, find the radius of the third sphere.

43. Calculate the co-efficient of variation of the following data 20, 18, 32, 24, 26.

44. A bag contains 10 white, 5 black, 3 green and 2 red balls. One ball is drawn at random. Find the probability that the ball drawn is white or black or green.

45. a) If $P = \frac{x}{x+y}$, $Q = \frac{y}{x+y}$, then find

$$\frac{1}{P-Q} - \frac{2Q}{P^2 - Q^2}$$

[OR]

b) D is the midpoint of the side BC of ΔABC. If P and Q are points on AB and on AC such that DP bisects $\angle BDA$ and DQ bisects $\angle ADC$, then prove that $PQ \parallel BC$

SECTION - IV

Note: Answer **both** the questions choosing either of the alternatives. **[2 × 10 = 20]**

46. (a) Draw a circle of radius 3 cm. From an external point 7 cm away from its centre, construct the pair of tangents to the circle and measure their lengths.

[OR]

(b) Construct a ΔABC such that BC = 5 cm, $\angle A = 45°$ and the median from A to BC is 4 cm

47. (a) Draw the graph of $y = 2x^2$ and hence solve $2x^2 + x - 6 = 0$.

[OR]

(b) The cost of milk per litre is ₹ 15. Draw the graph for the relation between the quantity and cost. Hence find :

(i) the proportionality constant.

(ii) the cost of 3 liters of milk.

★★★

ANSWERS

SECTION – I

1. (b)	2. (a)	3. (d)	4. (b)	5. (d)
6. (b)	7. (d)	8. (b)	9. (a)	10.(b)
11. (d)	12.(a)	13.(d)	14.(b)	15.(a)

SECTION – II

16. ***Solution:***

$B \cap C = \{2, 4, 6\} \cap \{1, 2, 3, 4, 5, 6\} = \{2, 4, 6\}$

$\therefore A \cup (B \cap C) = \{4, 6, 7, 8, 9\} \cup \{2, 4, 6\}$

$= \{2, 4, 6, 7, 8, 9\}$

17. ***Solution:***

Now, $A = \{ 1, 2, 3, 4, 5 \}$; $B = \{ 1, 2, 3, 4, ...\}$

Civen $f : A \to B$ and $f(x) = x^2$

$\therefore f(1) = 1^2 = 1;\ f(2) = 4;\ f(3) = 9;$

$f(4) = 16;\quad f(5) = 25.$

Range of $f = \{ 1, 4, 9, 16, 25\}$

Since distinct elements are mapped into distinct images, it is a one-one function.

However, the function is not onto, since $3 \in B$ but there is no $x \in A$ such that $f(x) = x^2 = 3$.

18. ***Solution:***

Let α and β be the zeros of a quadratic polynomial.

Given that $\alpha + \beta = -4$ and $\alpha\beta = 3$.

One of the such polynomials is

$p(x) = x^2 - (\alpha + \beta) + \alpha\beta$

$= x^2 - (-4)x + 3 = x^2 + 4x + 3.$

19. ***Solution :***

Since -5 is a root of $2x^2 + px - 15 = 0$

$2(-5)^2 + p(-5) - 15 = 0$

$\Rightarrow 2(25) - 5P - 15 = 0$

$\Rightarrow 50 - 5p - 15 = 0$

$\Rightarrow 35 - 5p = 0$

$\Rightarrow 35 = 5p \Rightarrow p = \frac{35}{5}$

$\Rightarrow p = 7$

20. ***Solution :***

$$C = 2A + B = 2\begin{pmatrix} 3 & 2 \\ 5 & 1 \end{pmatrix} + \begin{pmatrix} 8 & -1 \\ 4 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 4 \\ 10 & 2 \end{pmatrix} + \begin{pmatrix} 8 & -1 \\ 4 & 3 \end{pmatrix} = \begin{pmatrix} 14 & 3 \\ 14 & 5 \end{pmatrix}$$

21. ***Solution :***

$$\text{Now, } AI = \begin{pmatrix} 1 & 3 \\ 9 & -6 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1+0 & 0+3 \\ 9+0 & 0-6 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 9 & -6 \end{pmatrix} = A$$

$$\text{Also, } IA = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ 9 & -6 \end{pmatrix}$$

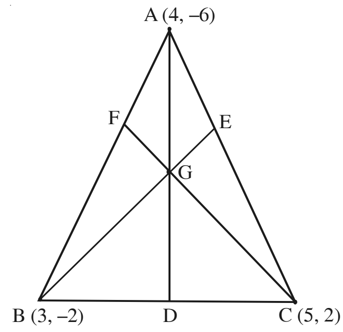
$$= \begin{pmatrix} 1+0 & 3+0 \\ 0+9 & 0-6 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 9 & -6 \end{pmatrix} = A$$

Hence $AI = IA = A$.

22. ***Solution:***

The centroid $G(x, y)$ of a triangle whose vertices are

(x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by



$$G(x, y) = G\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

we have $(x_1, y_1) = (4, -6)$, $(x_2, y_2) = (3, -2)$,

$(x_3, y_3) = (5, 2)$

$\therefore$ The centroid of the triangle whose vertices are (4, –6), (3, –2) and (5, 2) is

$$G(x, y) = G\left(\frac{4+3+5}{3}, \frac{-6-2+2}{3}\right)$$

$$G = (4, -2)$$

23. ***Solution:***

In ΔABC, DE || BC

$$\therefore \frac{AB}{DB} = \frac{AC}{EC} \text{ (By Thales theorem)}$$

$$\Rightarrow \frac{2x}{x-3} = \frac{2x+3}{x-2}$$

$$\Rightarrow 2x(x-2) = (2x+3)(x-3)$$

$$\Rightarrow \cancel{2x^2} - 4x = \cancel{2x^2} - 6x + 3x - 9$$

$$\Rightarrow -4x = -3x - 9 \qquad \Rightarrow -4x + 3x = -9$$

$$\Rightarrow -x = -9 \qquad \Rightarrow x = 9.$$

24. ***Solution:***

Now $\sin^6\theta + \cos^6\theta$

$= (\sin^2\theta)^3 + (\cos^2\theta)^3$

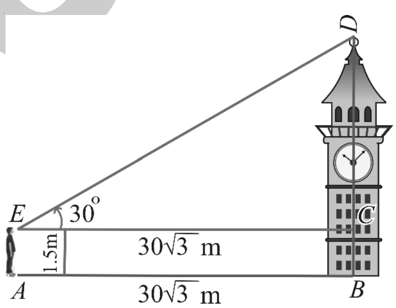
$= (\sin^2\theta + \cos^2\theta)^3 - 3\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta)$

$(a^3 + b^3 = (a+b)^3 - 3ab(a+b))$

$= 1 - 3\sin^2\theta\cos^2\theta. \quad (\sin^2\theta + \cos^2\theta = 1)$

25. ***Solution:***

Let BD be the height of the tower and *AE* be the distance of the eye level of the observer from the ground level.



Draw *EC* parallel to *AB* such that *AB* = *EC*. Given *AB* = *EC* = $30\sqrt{3}$ m and *AE* = *BC* = 1.5 m

In right angled ΔDEC

$$\tan 30^\circ = \frac{CD}{EC}$$

$$\Rightarrow CD = EC \tan 30^\circ = \frac{30\sqrt{3}}{\sqrt{3}}$$

$$\therefore CD = 30\text{m}$$

Thus, the height of the tower *BD* = *BC* + *CD* = 1.5 + 30 = 31.5m.

26. ***Solution:***

Ratio of radii $r_1 : r_2 = 2 : 3$

Ratio of heights $h_1 : h_2 = 5 : 3$

Ratio of volumes $= \cancel{\pi} r_1^2 h_1 : \cancel{\pi} r_2^2 h_2$

$$= 2^2 \times 5 : 3^2 \times 3 = 4 \times 5 : 9 \times 3$$

$$\Rightarrow V_1 : V_2 = 20 : 27$$

27. ***Solution:***

Circumference of the base of the cone = 236

$$\Rightarrow \cancel{2}(\pi r) = \cancelto{118}{236} \Rightarrow \pi r = 118$$

Slant height, l = 12 cm.

$$\therefore \text{C.S.A.} = (\pi r) l = 118 \times 12 = 1416\,\text{cm}^2.$$

28. ***Solution:***

$$x = 1 + 2 + 3 + \ldots\ldots + 13 = \frac{13 \times 14}{2} = 91$$

$$\Rightarrow \sum x = 91$$

Hint : $\sum n = \frac{n(n+1)}{2}$

$$\therefore \bar{x} = \frac{\sum x}{n} = \frac{91}{13} = 7$$

x	$d = x - \bar{x}$	d^2
1	–6	36
2	–5	25
3	–4	16
4	–3	9
5	–2	4
6	–1	1
7	0	0
8	1	1
9	2	4
10	3	9
11	4	16
12	5	25
13	6	36
		$\sum d^2 = 182$

Aliter Method :

S.D of the first n,

natural numbers = $\sqrt{\dfrac{n^2-1}{12}}$

$$n = 13 = \sqrt{\frac{13^2-1}{12}}$$

$$= \sqrt{\frac{169-1}{12}} = \sqrt{\frac{168}{12}}$$

$$= \sqrt{14} = 3.74.$$

$$\text{S.D.} = \sqrt{\frac{\sum d^2}{n}} = \sqrt{\frac{182}{13}} = \sqrt{14} = 3.74$$

29. ***Solution:***

S = {Three dice thrown simultaneously}

$\therefore$ n(S) = (6) (6) (6) = 216.

Let A be the event of getting the same number on all the 3 dice.

= { (1,1,1), (2,2,2), (3,3,3), (4,4,4), (5,5,5), (6,6,6)} $\Rightarrow$ n(A) = 6

$$\text{P(A)} = \frac{n(\text{A})}{n(\text{S})} = \frac{6}{216} = \frac{1}{36}$$

30. ***Solution:***

(a) 31 + 33 + ... + 53

=(1 + 3 + 5 +...+ 53) – (1 + 3 + 5 + ... + 29)

$$= \left(\frac{53+1}{2}\right)^2 - \left(\frac{29+1}{2}\right)^2$$

$$\left(1+3+5+\ldots+l = \left(\frac{l+1}{2}\right)^2\right)$$

$$= 27^2 - 15^2 = 504.$$

(b) Given that $m = \dfrac{1}{2}$, $c = 3$

By slope - intercept form $y = mx + c$

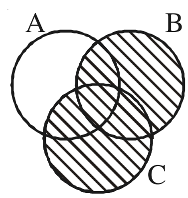
$$\Rightarrow y = \frac{x}{2} + 3 \Rightarrow x - 2y + 6 = 0.$$

Here the equation of the required line is $x - 2y + 6 = 0$.

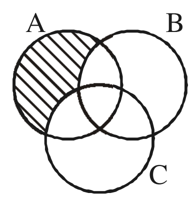
SECTION – III

31. ***Solution:***

$$\text{A} \setminus (\text{B} \cup \text{C}) = (\text{A} \setminus \text{B}) \cap (\text{A} \setminus \text{C})$$

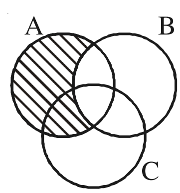


▧ $(\text{B} \cup \text{C})$

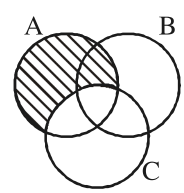


..(1)

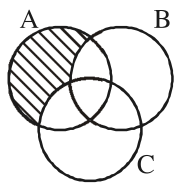
▧ $\text{A} \setminus (\text{B} \cup \text{C})$



▧ $(\text{A} \setminus \text{B})$



▧ $(\text{A} \setminus \text{C})$



..(2)

▧ $(A \setminus B) \cap (A \setminus C)$

From (1) & (2) it is clear that $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$

32. ***Solution:***

$A = \{ 0, 1, 2, 3 \}$, $B = \{ 1, 3, 5, 7, 9 \}$,

$$f(x) = 2x + 1$$

$f(0) = 2(0) + 1 = 1$, $f(1) = 2(1)+1 = 3$,

$f(2) = 2(2) + 1 = 5$, $f(3) = 2(3) + 1 = 7$

(i) Set of ordered pairs

The given function f can be represented as a set of ordered pairs as

$f = \{ (0, 1), (1, 3), (2, 5), (3, 7) \}$

(ii) Table form

Let us represent f using a table as shown below.

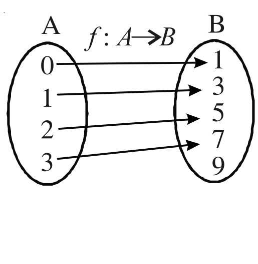
x	0	1	2	3
$f(x)$	1	3	5	7

(iii) Arrow Diagram

Let us represent f by an arrow diagram.

We draw two closed curves to represent the sets A and B.

Here each element of A and its unique image element in B are related with an arrow.



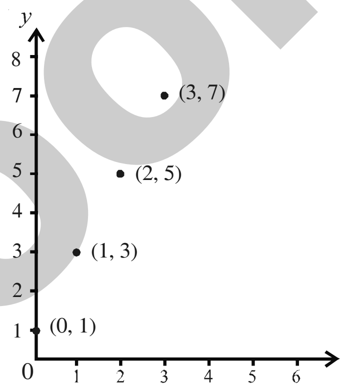
(iv) Graph

We are given that

$f = \{(x, f(x)) / x \in A\} = \{(0,1), (1, 3), (2, 5), (3, 7)\}$.

Now, the points (0, 1), (1, 3), (2, 5) and (3, 7) are plotted on the plane as shown below.

The totality of all points represent the graph of the function.



33. ***Solution:***

The natural numbers between 300 and 500 which are divisible by 11 are; 308, 319, ...495.

> ***Hint :***
>
> 11) 300 (27
> 22
> ---
> 80
> 77
> ---
> 3 → Remainder
>
> $\therefore$ 300 – 3 = 297
>
> 297 is exactly ÷ by 11
>
> $\Rightarrow$ 297 + 11 = 308
>
> 11) 500 (45
> 44
> ---
> 60
> 55
> ---
> 5 → Remainder
>
> $\therefore$ 500 – 5 = 495

$a = 308, \quad l = 495, \quad d = 11$

$$S_n = \frac{n}{2}\{a + l\}$$

$$\text{but } n = \frac{l-a}{d} + 1$$

$$S_{18} = \frac{18}{2}\{308 + 495\}$$

$$= \frac{495 - 308}{11} + 1$$

$$= 9\{803\} = 7227.$$

$$= \frac{187}{11} + 1 = 18$$

$\therefore$ The sum of all natural numbers between 300 and 500 which are divisible by 11 is 7227.

34. ***Solution:***

Let the G.P be $a, ar, ar^2, ...$

Given $t_4 = 54 \Rightarrow ar^{4-1} = 54 \Rightarrow ar^3 = 54$ ..... (1)

and $t_7 = 1458 \Rightarrow ar^{7-1} = 1458 \Rightarrow ar^6 = 1458$ ..... (2)

$$\frac{(2)}{(1)} \text{ gives } \frac{ar^6}{ar^3} = \frac{\overset{27}{\cancel{1458}}}{\cancel{54}} = 27$$

$\therefore r^3 = 27 \Rightarrow r = 3.$

Sub. in (1) $a(3)^3 = 54$

$$\therefore a = \frac{54}{3^3} = \frac{54}{27} = 2$$

Hence the G.P. is $2, 2(3), 2(3)^2, \ldots$ (i.e.) $2, 6, 18, \ldots$..

35. ***Solution:***

Let $p(x) = x^3 - 3x^2 - 10x + 24$

Since $p(1) \neq 0$ and $p(-1) \neq 0$, neither $x + 1$ nor $x - 1$ is a factor of $p(x)$.

Therefore, we have to search for different values of x by trial and error method.

When $x = 2$, $p(2) = 0$

Thus, $x - 2$ is a factor of $p(x)$.

To find the other factors, let us use the synthetic division.

$$\begin{array}{c|cccc} 2 & 1 & -3 & -10 & 24 \\ & 0 & 2 & -2 & -24 \\ \hline & 1 & -1 & -12 & \underline{|0} \end{array} \rightarrow \text{Remainder.}$$

$\therefore$ The other factor is $x^2 - x - 12$.

Now, $x^2 - x - 12 = x^2 - 4x + 3x - 12$

$= (x - 4)(x + 3)$

Hence, $x^3 - 3x^2 - 10x + 24 = (x - 2)(x + 3)(x - 4)$

36. ***Solution:***

$4x^4 - 12x^3 + 37x^2 + ax + b$

Hint :

$4x^4 = (2x^2)^2$

$\dfrac{-12x^3}{4x^2} = -3x$

$\dfrac{28x^2}{4x^2} = 7$

$ax + 42x = 0 \qquad b - 49 = 0$

$a\cancel{x} = -42\cancel{x} \qquad b = 49$

$a = -42$

$$\begin{array}{r|l} & \quad 2x^2 - 3x + 7 \\ \hline 2x^2 & 4x^4 - 12x^3 + 37x^2 + ax + b \\ & 4x^4 \\ \hline 4x^2 - 3x & \quad -12x^3 + 37x^2 \\ & \quad -12x^3 + 9x^2 \\ \hline 4x^2 - 6x + 7 & \qquad 28x^2 + ax + b \\ & \qquad 28x^2 - 42x + 49 \\ \hline & \qquad\qquad 0 \end{array}$$

Since the given polynomial is a perfect square, we must have $a = -42$ and $b = 49$.

37. ***Solution:***

$$A^2 = A.A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1-2 & -1-3 \\ 2+6 & -2+9 \end{pmatrix} = \begin{pmatrix} -1 & -4 \\ 8 & 7 \end{pmatrix}$$

$$-4A = -4\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} -4 & 4 \\ -8 & -12 \end{pmatrix};$$

$$5I_2 = 5\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$$

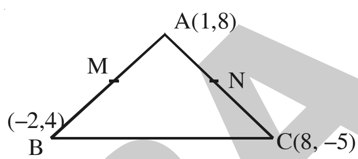
$A^2 - 4A + 5I_2$

$$= \begin{pmatrix} -1 & -4 \\ 8 & 7 \end{pmatrix} + \begin{pmatrix} -4 & 4 \\ -8 & -12 \end{pmatrix} + \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} -1-4+5 & -4+4+0 \\ 8-8+0 & 7-12+5 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = 0$$

$$A^2 - 4A + 5I_2 = 0$$

38. ***Solution:***



$$\text{Mid-point of AB} = M\left(\frac{-2+1}{2}, \frac{4+8}{2}\right)$$

$$= M\left(\frac{-1}{2}, 6\right)$$

$$\text{Mid-point of AC} = N\left(\frac{1+8}{2}, \frac{8-5}{2}\right)$$

$$= N\left(\frac{9}{2}, \frac{3}{2}\right)$$

$$\therefore \text{slope of MN} = \frac{6-\frac{3}{2}}{\frac{-1}{2}-\frac{9}{2}} = \frac{\frac{12-3}{2}}{\frac{-1-9}{2}} = \frac{\frac{9}{2}}{-\frac{10}{2}} = -\frac{9}{10}$$

$$\text{slope of BC} = \frac{4+5}{-2-8} = \frac{9}{-10} = -\frac{9}{10}.$$

Hence MN is parallel to BC.

[Since slope of MN = Slope of BC.]

39. ***Solution:***

Let the given line intersects the x-axis and y-axis at A and B.

Put $y = 0$, to get the length of x - intercept

$\therefore 4x + 3(0) - 12 = 0$

$\Rightarrow 4x = 12 \Rightarrow x = \frac{12}{4} = 3$

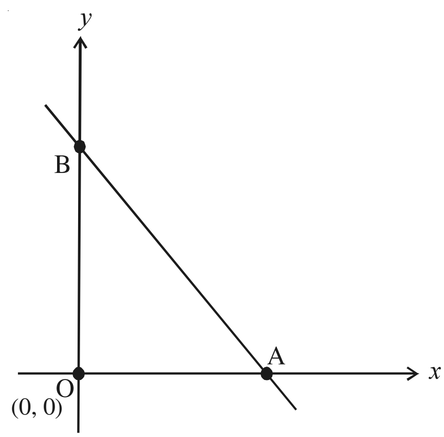
$\Rightarrow$ OA = 3

Put $x = 0$, to get the length of y - intercept

$4(0) + 3y - 12 = 0 \Rightarrow 3y = 12$

$\Rightarrow y = \frac{12}{3} = 4$

$\therefore$ OB = 4.



Area of $\Delta OAB = \frac{1}{2} \times OA \times OB$

[Since ΔBC is an right angled triangle]

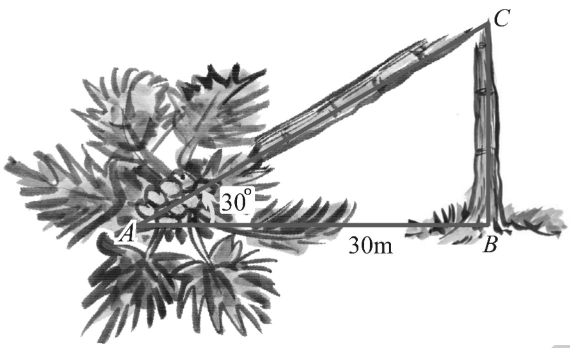
$= \frac{1}{2} \times 3 \times 4 = 6$ sq. units.

40. ***Solution:***

Let C be the point at which the tree is broken and let the top of the tree touch the ground at A.

Let B denote the foot of the tree.

Given AB = 30 m and $\angle CAB = 30^o$.



In the right angled ΔCAB,

$$\tan 30^o = \frac{BC}{AB}$$

$$\Rightarrow BC = AB \tan 30^o$$

$$\therefore \; BC = \frac{30}{\sqrt{3}} = 10\sqrt{3} \text{ m} \quad \ldots (1)$$

Now, $\cos 30^o = \dfrac{AB}{AC} \Rightarrow AC = \dfrac{AB}{\cos 30^o}$

So, $AC = \dfrac{30 \times 2}{\sqrt{3}} = 10\sqrt{3} \times 2 = 20\sqrt{3}$ m...(2)

Thus, the height of the tree

$= BC + AC = 10\sqrt{3} + 20\sqrt{3} = 30\sqrt{3}$ m.

41. ***Solution:***

Cone	Cylinder
$r = 5$ cm	$r = 5$ cm
$h = 12$ cm	$h = 80 - 12 = 68$ cm.

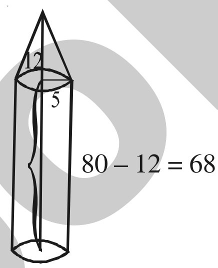
Total surface area of the circket stemp

$= 2\pi rh + \pi rl$

$= 2 \times \dfrac{22}{7} \times 5 \times 68 + \dfrac{22}{7} \times 5 \times l$

$l = \sqrt{h^2 + r^2} = \sqrt{12^2 + 5^2}$

$= \sqrt{144 + 25 = 13}$



$\therefore$ Total surface area of the circket stemp

$= \dfrac{22}{7} \times 5 \; [2 \times 68 + 13]$

$= \dfrac{22}{7} \times 5 \; [136 + 13] = \dfrac{22}{7} \times 5 \times 149$

$= 2341.43$ sq. m.

42. ***Solution:***

Volume of the solid sphere

melted $= \dfrac{4}{3}\pi \times 18 \times 18 \times 18$

Volume of spheres

made $= \dfrac{4}{3}\pi \; [2^3 + 12^3 + r^3]$

Volume melted = Volume recast.

Hint:

4	4096
4	1024
4	256
4	64
4	16
4	4

$$\therefore \frac{4}{3}\pi[8+1728+r^3] = \frac{4}{3}\pi \times 18\times 18\times 18$$

$$1736 + r^3 = 5832$$

$$r^3 = 4096$$

$$= 4^3 \times 4^3$$

$$\therefore r = 4\times 4 = 16 \text{ cm}$$

43. ***Solution:***

x	$d = x-24$	d^2
20	–4	16
18	–6	36
32	8	64
24	0	0
26	2	4
		$\sum d^2 = 120$

$$\sum x = 120$$

$$\therefore \quad \bar{x} = \frac{\sum x}{n} = \frac{120}{5} = 24$$

$$\sigma = \sqrt{\frac{\sum d^2}{n}} = \sqrt{\frac{120}{5}} = \sqrt{24} = 4.89$$

$$\text{C.V} = \frac{\sigma}{\bar{x}}\times 100 = \frac{4.89}{24}\times 100 = 20.375 = 20.41$$

44. ***Solution:***

Let S be the sample space. $\therefore n$ (S) = 20.

Let W, B and G be the events of selecting a white, black and green ball respectively.

Probability of getting a white ball, P (W)

$$= \frac{n(\text{W})}{n(\text{S})} = \frac{10}{20}.$$

Probability of getting a black ball, P (B)

$$= \frac{n(\text{B})}{n(\text{S})} = \frac{5}{20}.$$

Probability of getting a green ball,

$$\text{P (G)} = \frac{n(\text{G})}{n(\text{S})} = \frac{3}{20}.$$

$\therefore$ Probability of getting a white or black or green ball,

P (W ∪ B ∪ G) = P (W) + P (B) + P (G)

$\because$ W, B and G are mutually exclusive.

$$= \frac{10}{20} + \frac{5}{20} + \frac{3}{20} = \frac{9}{10}$$

45. (a) ***Solution:***

$$\frac{1}{P-Q} - \frac{2Q}{P^2 - Q^2}$$

$$= \frac{1}{\frac{x}{x+y} - \frac{y}{x+y}} - \frac{\frac{2y}{x+y}}{\frac{x^2}{(x+y)^2} - \frac{y^2}{(x+y)^2}}$$

$$= \frac{(x+y)}{x-y} - \frac{\frac{2y}{x+y}}{\frac{x^2-y^2}{(x+y)^2}}$$

$$= \frac{x+y}{x-y} - \frac{2y}{(x+y)} \times \frac{(x+y)^2}{(x-y)(x+y)}$$

$$= \frac{x+y}{x-y} - \frac{2y}{x-y}$$

$$= \frac{x+y-2y}{(x-y)} = \frac{(x-y)}{(x-y)} = 1$$

Aliter :

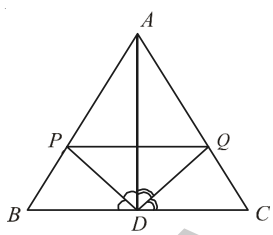
$$\frac{1}{P-Q}-\frac{2Q}{P^2-Q^2}=\frac{1}{P-Q}-\frac{2Q}{(P+Q)(P-Q)}$$

$$=\frac{1}{P-Q}\left[\frac{1}{1}-\frac{2Q}{P+Q}\right]-\frac{1}{P-Q}\left[\frac{P+Q-2Q}{P+Q}\right]$$

$$=\frac{1}{\cancel{P-Q}}\left[\frac{\cancel{(P-Q)}}{(P+Q)}\right]=\frac{1}{P+Q}$$

$$=\frac{1}{\frac{x}{x+y}+\frac{y}{x+y}}=\frac{1}{\frac{\cancel{x+y}}{\cancel{x+y}}}=1$$

(b) In ΔABD, DP is the angle bisector of $\angle BDA$.



$\therefore \frac{AP}{PB}=\frac{AD}{BD}$ (angle bisector theorem) — (1)

In ΔADC, DQ is the bisector of $\angle ADC$

$\therefore \frac{AQ}{QC}=\frac{AD}{DC}$ (angle bisector theorem)— (2)

But, BD = DC (D is the midpoint of BC)

Now (2) $\Rightarrow \frac{AQ}{QC}=\frac{AD}{BD}$ —— (3)

From (1) and (3) we get, $\frac{AP}{PB}=\frac{AQ}{QC}$

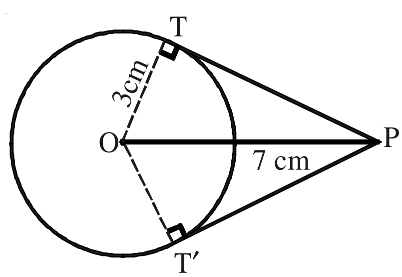
Thus, PQ || BC(converse of Thales theorem)

46. ***Solution:***

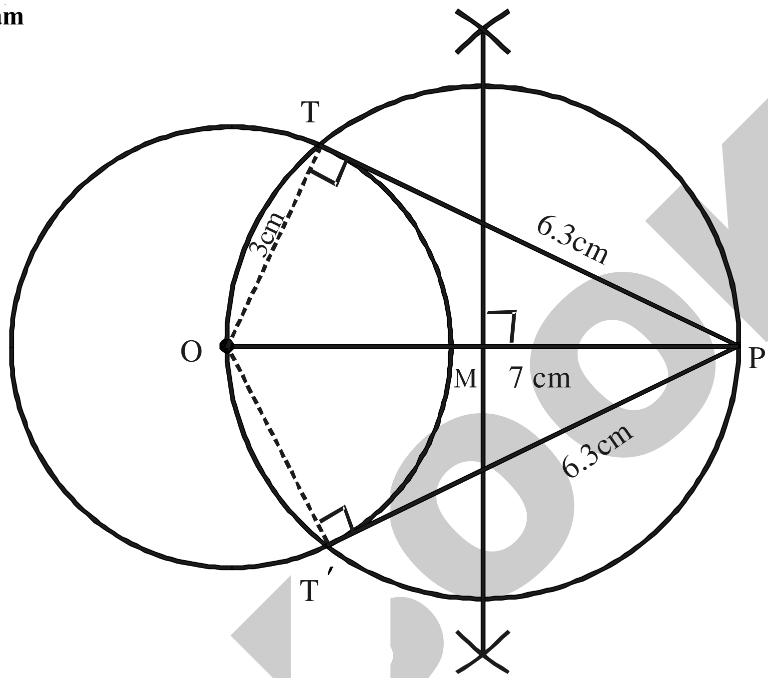
(a) **Given:**

Radius of the circle = 3cm. OP = 7cm.

Rough Diagram



Fair Diagram



Construction:

(i) With O as the centre draw a circle of radius 3 cm.

(ii) Mark a point P at a distance of 7cm from O and join OP.

(iii) Draw the perpendicular bisector of OP. Let in meet OP at M.

(iv) with M as centre and MO as radius, draw another circle.

(v) Let the two circles intersect at T and T′

(vi) Join PT and P′. They are the required tangents.

Length of the tangent, PT = 6.3 cm

Verification:

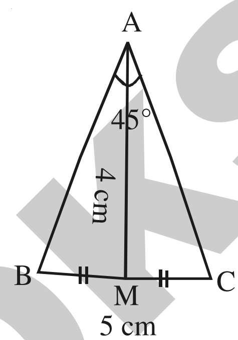
In the right angled $\triangle$ OPT,

$$PT = \sqrt{OP^2 - OT^2} = \sqrt{7^2 - 3^2}$$

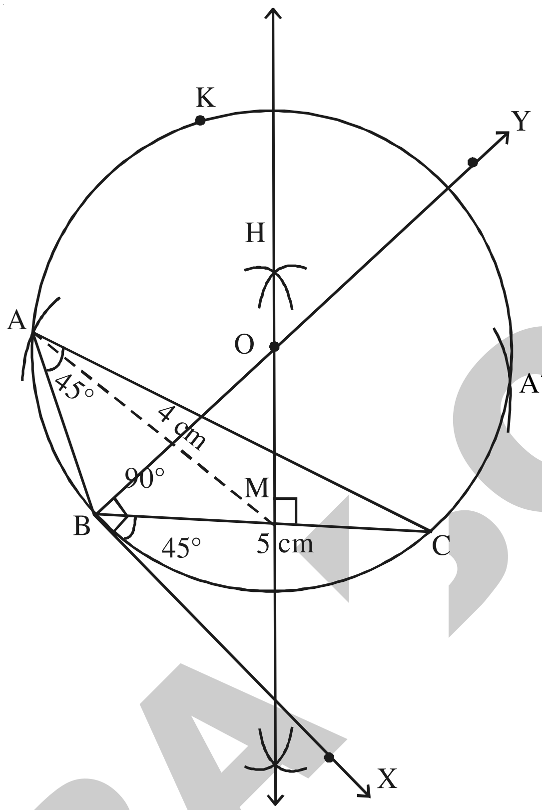
$$= \sqrt{49 - 9} = \sqrt{40} \quad \therefore \; PT = 6.3\text{cm (approximately)}$$

46. (b)

Rough Diagram



Fair Diagram



Steps of Construction:

(i) Draw a line segment BC = 5 cm.

(ii) Through B draw BX such that $\angle CBX = 45^{\circ}$

(iii) Draw $BY \perp BX$.

(iv) Draw the perpendicular bisector of BC intersecting BY at O and BC at M.

(v) With O as centre and OB as radius, draw the circle.

(vi) The major arc BKC of the circle, contains the vertical angle 45º.

(vii) With M as centre, draw an arc of radius 4 cm meeting the circle at A and A'.

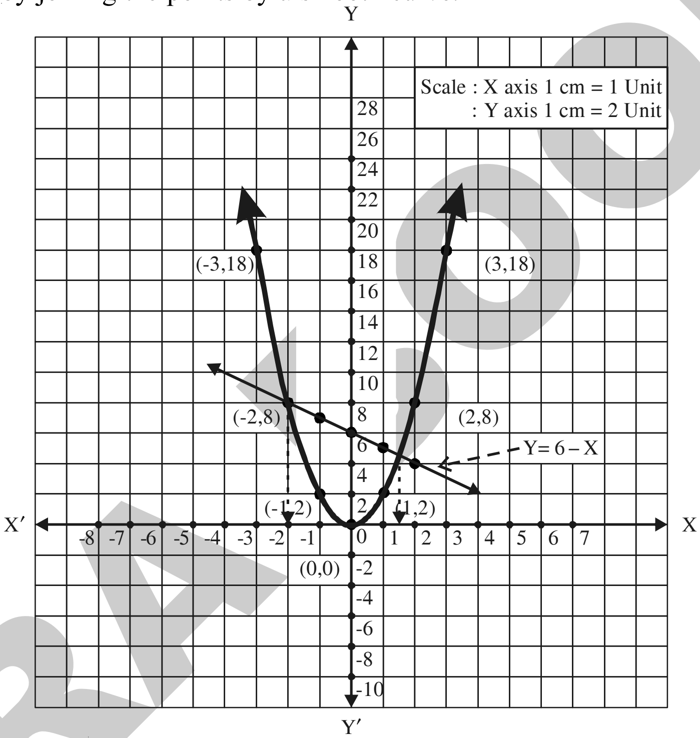
(viii) Δ ABC or Δ A'BC is the required triangle.

47. (a) ***Solution:***

First, let us draw the graph of $y=2x^2$. From the following table.

x	–3	–2	–1	0	1	2	3
x^2	9	4	1	0	1	4	9
$y = 2x^2$	18	8	2	0	2	8	18

Plot the points (– 3, 18), (– 2, 8), (– 1, 2), (0, 0), (1, 2), (2, 8), (3, 18).
Draw the graph by joining the points by a smooth curve.



To find the roots of $2x^2+x-6=0$, solve the two equations $y=2x^2$ and $2x^2+x-6=0$.
Now, $2x^2+x-6=0 \Rightarrow y+x-6=0$, since $y=2x^2$
Thus, $y=-x+6$
Hence, the roots of $2x^2+x-6=0$ are nothing but the x - coordinates of the points of intersection of $y=2x^2$ and $y=-x+6$.
Now, for the staight line $y=-x+6$, form the following table.

x	–1	0	1	2
$y = -x + 6$	7	6	5	4

Draw the straight line by joining the above points. The points of intersection of the line and the parabola are (–2, 8) and (1.5, 4.5), The x-coordinates of the points are –2 and 1.5. Thus, the solution set for the equation $2x^2+x-6=0$ is {–2, 1.5}.

47. (b) ***Solution:***

Let us form the following table.

Quantity of milk in litres x	1	2	3	4	5
Cost per litre in ₹ y	15	30	45	60	75

From the table we observe that as x increases, y also increases. Therefore it is in direct variation.

$\therefore$ we get $y \propto x$ (i.e.) $y = kx$ where k is a constant of proportionality.

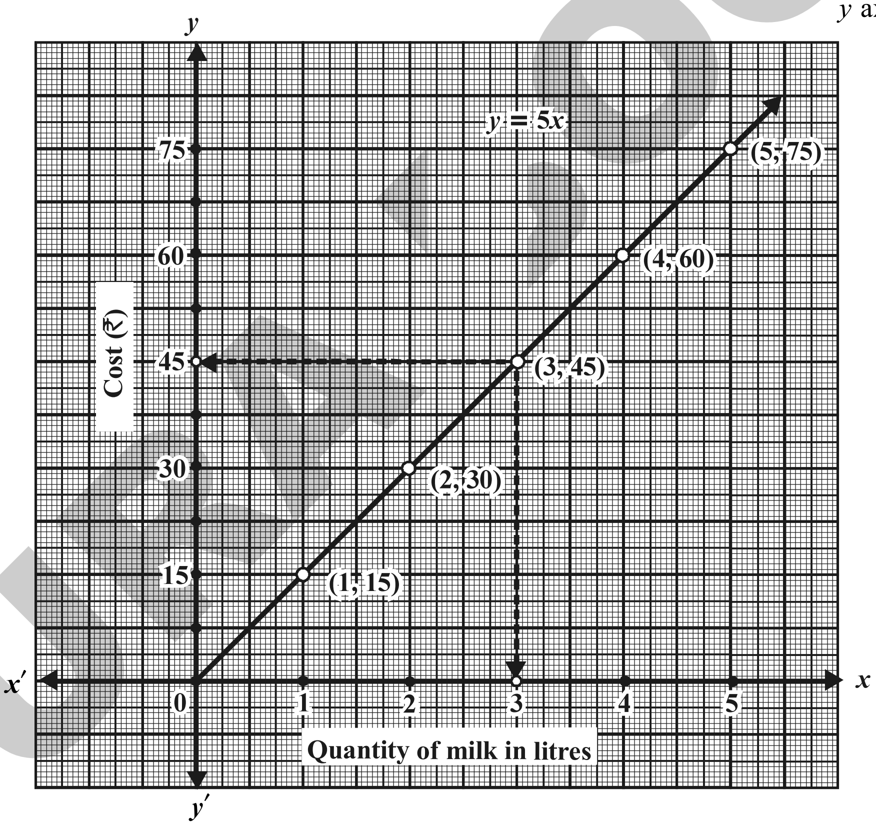
Since $y/x = k$, From the table we find $15/1 = 30/2 = 45/3 = 60/4 = 75/5 = k = 15$.

$\therefore$ we get $k = 15$.

Plot the points (1, 15), (2, 30), (3, 45, (4, 60) and (5, 75) and join them.

Scale: x axis 2 cm = 1 litre

y axis 2 cm = ₹ 15



$\therefore$ The relation $y = 15\,x$ is a straight line as exhibited is the graph.

From the graph we find,

(i) The proportionality constant $k = 15$

(ii) The cost of 3 litres of milk = ₹ 45

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